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A precision constraint on multi-Higgs-doublet models

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Abstract

We derive a general expression for $\Delta\rho$ (or, equivalently, for the oblique parameter T) in the $SU(2)\times U(1)$ electroweak model with an arbitrary number of scalar $SU(2)$ doublets, with hypercharge $\pm 1/2$, and an arbitrary number of scalar $SU(2)$ singlets. The experimental bound on $\Delta\rho$ constitutes a strong constraint on the masses and mixings of the scalar particles in that model.

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1 Introduction

In the Standard Model (SM), the parameter

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}, \tag{1}$$

where m_W and m_Z are the masses of the $W^\pm$ and Z^0 gauge bosons, respectively, and θ_W is the weak mixing angle, gives the relative strength of the neutral-current and charged-current interactions in four-fermion processes at zero momentum transfer [1]. At tree level ρ is equal to one, and it remains one even if additional scalar $SU(2)$ doublets, with hypercharge $\pm 1/2$, are added to the SM.[1] At one-loop level, the vacuum-polarization effects, which are sensitive to any field that couples either to the $W^\pm$ or to the Z^0, produce the vacuum-polarization tensors ($V = W, Z$)

$$\Pi_{VV}^{\mu\nu}(q) = g^{\mu\nu} A_{VV}\left(q^2\right) + q^\mu q^\nu B_{VV}\left(q^2\right), \tag{2}$$

where q^μ is the four-momentum of the gauge boson. Then, deviations of ρ from unity arise, which are determined by the self-energy difference [1, 2]

$$\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}. \tag{3}$$

The precise measurement [3], at LEP, of the $W^\pm$ and Z^0 self-energies is in striking agreement with the SM predictions [4] and provides a strong constraint on extended electroweak models. For instance, one can constrain the two-Higgs-doublet model (2HDM) in this way [5, 6].

In this paper we are interested in the contributions to the ρ parameter generated by an extension of the SM. Therefore, we define a $\Delta\rho$ which refers to the non-SM part of the quantity (3):

$$\Delta\rho = \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}\right]_{\text{SM extension}} - \left[\frac{A_{WW}(0)}{m_W^2} - \frac{A_{ZZ}(0)}{m_Z^2}\right]_{\text{SM}}. \tag{4}$$

The SM contributions to the quantity (3) are known up to the leading terms at three-loop level [7]. However, the consistent SM subtraction in equation (4) only requires the one-loop SM result. In the same vein, we are allowed to make the replacement $m_Z^2 = m_W^2/c_W^2$ in equation (4), writing instead

$$\Delta\rho = \left[\frac{A_{WW}(0) - c_W^2 A_{ZZ}(0)}{m_W^2}\right]_{\text{SM extension}} - \left[\frac{A_{WW}(0) - c_W^2 A_{ZZ}(0)}{m_W^2}\right]_{\text{SM}}. \tag{5}$$

Here and in the following, we use the abbreviations $c_W = \cos\theta_W$, $s_W = \sin\theta_W$.

At one loop, the contributions of new physics to the self-energies constitute intrinsically divergent Feynman diagrams, but the divergent parts cancel out among different diagrams,

[1] Other scalar $SU(2) \times U(1)$ representations are also allowed, as long as they have vanishing vacuum expectation values.

between $A_{WW}(0)$ and $c_W^2 A_{ZZ}(0)$, and also, eventually, through the subtraction of the SM contributions laid out in equation (5). If the new-physics model is renormalizable, then $\Delta\rho$ is finite. The cancellations finally leave either a quadratic or a logarithmic dependence of $\Delta\rho$ on the masses of the new-physics particles. The pronounced effects of large masses is what renders the parameter $\Delta\rho$ so interesting for probing physics beyond the Standard Model.

The functions $A_{VV}(q^2)$ contain more information about new physics than the one just provided by $\Delta\rho$. In fact, for new physics much above the electroweak scale, a detailed analysis of the so-called "oblique corrections" lead to the identification of three relevant observables, which were called S, T and U in [8] and ϵ_1, ϵ_2 and ϵ_3 in [9].[2] While these two sets of observables differ in their precise definitions, the quantity of interest in this paper is simply

$$\Delta\rho = \alpha T = \epsilon_1, \tag{6}$$

where $\alpha = e^2/(4\pi) = g^2 s_W^2/(4\pi)$ is the fine-structure constant.

It is not straightforward to obtain a bound on $\Delta\rho$ from electroweak precision data. One possibility is to add the oblique parameters to the SM parameter set and perform fits to the data. However, since the SM Higgs-boson loops themselves resemble oblique effects, one cannot determine the SM Higgs-boson mass m_h simultaneously with S and T [4]. To get a feeling for the order of magnitude allowed for $\Delta\rho$, we quote the number

$$T = -0.03 \pm 0.09\ (+0.09), \tag{7}$$

which was obtained in [4] by fixing $U = 0$. For the mean value of T, the Higgs-boson mass $m_h = 117$ GeV was assumed; the mean value in parentheses is for $m_h = 300$ GeV. Equation (7) translates into $\Delta\rho = -0.0002 \pm 0.0007\ (+0.0007)$.

There is a vast literature on the 2HDM—see [11] for a review, [12] for the renormalization of the model, [13, 14] for the possibility of having a light pseudoscalar compatible with all experimental constraints, and [15, 16], and the references therein, for other various recent works. However, just as the 2HDM may differ significantly from the SM, a general multi-Higgs-doublet model may be quite different from its minimal version with only two Higgs doublets [17]. Three or more Higgs doublets frequently appear in models with family symmetries through which one wants to explain various features of the fermion masses and mixings; for some examples in the lepton sector see the reviews in [18].

In this paper we present a calculation of $\Delta\rho$ in an extension of the SM with an arbitrary number of Higgs doublets and also, in addition, arbitrary numbers of neutral and charged scalar $SU(2)$ singlets. Our results can be used to check the compatibility of the scalar sector of multi-Higgs models with the constraints resulting from the electroweak precision experiments.

Recently, there has been some interest in "dark" scalars [19, 20]. These are scalars that have no Yukawa couplings, and are thus decoupled from ordinary matter. Furthermore, they have no vacuum expectation values (VEVs) and therefore display truncated couplings to the gauge bosons. However, they would have quadrilinear vector–vector–scalar–scalar

[2] For new physics at a mass scale comparable to the electroweak scale three more such "oblique parameters" have been identified in [10].

and trilinear vector–scalar–scalar (but no vector–vector–scalar) couplings, and would thus also contribute to, and be constrained by, $\Delta\rho$.

The plan of the paper is as follows. In Section 2 we present a description of our extension of the SM and the final result of the calculation of $\Delta\rho$; this section is self-consistent and the result can be used without need to consult the rest of the paper. The details of the calculation are laid out in Section 3. The application of our $\Delta\rho$ formula to the general 2HDM is given in Section 4. The summary of our study is found in Section 5.

2 The model and the result for $\Delta\rho$

2.1 The model

We consider an $SU(2) \times U(1)$ electroweak model in which the scalar sector includes n_d $SU(2)$ doublets with hypercharge $1/2$,[3]

$$\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix}, \quad k = 1, 2, \ldots, n_d. \tag{8}$$

Moreover, we allow the model to include an arbitrary number and variety of $SU(2)$-singlet scalars; in particular, n_c complex $SU(2)$ singlets with hypercharge 1,

$$\chi_j^+, \quad j = 1, 2, \ldots, n_c \tag{9}$$

and n_n real $SU(2)$ singlets with hypercharge 0,

$$\chi_l^0, \quad l = 1, 2, \ldots, n_n. \tag{10}$$

In general, our model may include other scalar fields, singlet under the gauge $SU(2)$, with different electric charges.

The neutral fields are allowed to have vacuum expectation values (VEVs). Thus,

$$\left\langle 0 \left| \varphi_k^0 \right| 0 \right\rangle = \frac{v_k}{\sqrt{2}}, \tag{11}$$

$$\left\langle 0 \left| \chi_l^0 \right| 0 \right\rangle = u_l, \tag{12}$$

the v_k being in general complex. (The u_l are real since the χ_l^0 are real fields.) We define as usual $v = \left(\sum_{k=1}^{n_d} |v_k|^2 \right)^{1/2} \simeq 246\,\mathrm{GeV}$. Then, the masses of the $W^\pm$ and Z^0 gauge bosons are, at tree level, $m_W = gv/2$ and $m_Z = m_W/c_W$, respectively.[4] We expand the neutral fields around their VEVs,

$$\varphi_k^0 = \frac{1}{\sqrt{2}} \left(v_k + \varphi_k^{0\prime} \right), \tag{13}$$

[3]Equivalently, we may consider the model to contain $SU(2)$ doublets with hypercharge $-1/2$, since

$$\tilde{\phi}_k \equiv i\tau_2 \phi_k^* = \begin{pmatrix} \varphi_k^{0\,*} \\ -\varphi_k^- \end{pmatrix}$$

is also a doublet of $SU(2)$.

[4]Since the neutral singlet fields carry no hypercharge, their VEVs u_l do not contribute to the masses of the gauge bosons.

$$\chi_l^0 = u_l + \chi_l^{0\prime}. \tag{14}$$

Altogether, there are $n = n_d + n_c$ complex scalar fields with electric charge 1 and $m = 2n_d + n_n$ real scalar fields with electric charge 0. The mass matrices of all these scalar fields will in general lead to their mixing. The physical (mass-eigenstate) charged and neutral scalar fields will be called S_a^+ ($a = 1, 2, \ldots, n$) and S_b^0 ($b = 1, 2, \ldots, m$), respectively. Note that the fields S_b^0 are real. We use m_a to denote the mass of $S_a^\pm$ and μ_b to denote the mass of S_b^0. We have

$$\varphi_k^+ = \sum_{a=1}^{n} U_{ka} S_a^+, \tag{15}$$

$$\chi_j^+ = \sum_{a=1}^{n} T_{ja} S_a^+, \tag{16}$$

$$\varphi_k^{0\prime} = \sum_{b=1}^{m} V_{kb} S_b^0, \tag{17}$$

$$\chi_l^{0\prime} = \sum_{b=1}^{m} R_{lb} S_b^0, \tag{18}$$

the matrices U, T, V and R having dimensions $n_d \times n$, $n_c \times n$, $n_d \times m$ and $n_n \times m$, respectively. The matrix R is real, the other three are complex. The matrix

$$\tilde{U} \equiv \begin{pmatrix} U \\ T \end{pmatrix} \tag{19}$$

is $n \times n$ unitary; it is the matrix which diagonalizes the (Hermitian) mass matrix of the charged scalars. The real matrix

$$\tilde{V} \equiv \begin{pmatrix} \mathrm{Re}\, V \\ \mathrm{Im}\, V \\ R \end{pmatrix} \tag{20}$$

is $m \times m$ orthogonal; it diagonalizes the (symmetric) mass matrix of the real components of the neutral-scalar fields.[5]

There are in the spontaneously broken $SU(2)\times U(1)$ theory three unphysical Goldstone bosons, $G^\pm$ and G^0. For definiteness we assign to them the indices $a = 1$ and $b = 1$, respectively:

$$S_1^\pm = G^\pm, \tag{21}$$

$$S_1^0 = G^0. \tag{22}$$

Thus, only the $S_a^\pm$ with $a \geq 2$ are physical and, similarly, only the S_b^0 with $b \geq 2$ correspond to true particles. In the general 't Hooft gauge that we shall use in our computation, the masses of $G^\pm$ and G^0 are arbitrary and unphysical, and they cannot appear in the final result for $\Delta\rho$.

[5]Our treatment of the mixing of scalars is inspired by [21].

2.2 The result

As we shall demonstrate in the next section, the value of $\Delta\rho$ in the model outlined above is

$$\Delta\rho \;=\; \frac{g^2}{64\pi^2 m_W^2}\left\{\sum_{a=2}^{n}\sum_{b=2}^{m}\left|\left(U^\dagger V\right)_{ab}\right|^2 F\left(m_a^2,\mu_b^2\right)\right. \tag{23a}$$

$$-\sum_{b=2}^{m-1}\sum_{b'=b+1}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{bb'}\right]^2 F\left(\mu_b^2,\mu_{b'}^2\right) \tag{23b}$$

$$-2\sum_{a=2}^{n-1}\sum_{a'=a+1}^{n}\left|\left(U^\dagger U\right)_{aa'}\right|^2 F\left(m_a^2,m_{a'}^2\right) \tag{23c}$$

$$+3\sum_{b=2}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{1b}\right]^2\left[F\left(m_Z^2,\mu_b^2\right)-F\left(m_W^2,\mu_b^2\right)\right] \tag{23d}$$

$$\left.-3\left[F\left(m_Z^2,m_h^2\right)-F\left(m_W^2,m_h^2\right)\right]\right\}, \tag{23e}$$

where m_a, $m_{a'}$ denote the masses of the charged scalars and μ_b, $\mu_{b'}$ denote the masses of the neutral scalars. The term (23b) contains a sum over all pairs of *different* physical neutral scalar particles S_b^0 and $S_{b'}^0$; similarly, the term (23c) contains a sum over all pairs of different charged scalars, excluding the Goldstone bosons $G^\pm$, i.e. $2 \le a < a' \le n$. The term (23e) consists of the subtraction, from the rest of $\Delta\rho$, of the SM result—m_h is the mass of the sole SM physical neutral scalar, the so-called Higgs particle.

In equation (23), the function F of two non-negative arguments x and y is

$$F\left(x,y\right) \equiv \begin{cases} \dfrac{x+y}{2} - \dfrac{xy}{x-y}\ln\dfrac{x}{y} & \Leftarrow\; x \neq y, \\ 0 & \Leftarrow\; x = y. \end{cases} \tag{24}$$

This is a non-negative function, symmetrical under the interchange of its two arguments, and vanishing if and only if those two arguments are equal. This function has the important property that it grows linearly with $\max(x,y)$, i.e. quadratically with the heaviest-scalar mass, when that mass becomes very large. Unless there are cancellations, this leads to a quadratic divergence of $\Delta\rho$ for very heavy scalars (Higgs bosons).

If there are in the model any $SU(2)$-singlet scalars with electric charge other than 0 or ± 1, then the existence of those scalars does not contribute to $\Delta\rho$, they do not modify equation (23), at one-loop level, in any way.

A simplification occurs when there are in the model no $SU(2)$-singlet charged scalars χ_j^+. In that case, there is no matrix T, hence the matrix U is unitary by itself, and the term (23c) vanishes.

When there are in the model no $SU(2)$-singlet neutral scalars χ_l^0, there is no matrix R, hence $\mathrm{Re}\left(V^\dagger V\right)_{bb'} = \left(\mathrm{Re}\,V^T\ \mathrm{Re}\,V + \mathrm{Im}\,V^T\ \mathrm{Im}\,V\right)_{bb'} = \delta_{bb'}$. Then, in the terms (23b) and (23d) one may write $\left|\left(V^\dagger V\right)_{bb'}\right|^2$ instead of $\left[\mathrm{Im}\left(V^\dagger V\right)_{bb'}\right]^2$.

Thus, in an n_d-Higgs-doublet model *without any scalar singlets*, one has simply

$$\Delta\rho \;=\; \frac{g^2}{64\pi^2 m_W^2}\left\{\sum_{a=2}^{n_d}\sum_{b=2}^{2n_d}\left|\left(U^\dagger V\right)_{ab}\right|^2 F\left(m_a^2,\mu_b^2\right)\right.$$

$$
\begin{aligned}
&-\sum_{b=2}^{2n_d-1}\sum_{b'=b+1}^{2n_d}\left|\left(V^\dagger V\right)_{bb'}\right|^2 F\left(\mu_b^2,\mu_{b'}^2\right)\\
&+3\sum_{b=2}^{2n_d}\left|\left(V^\dagger V\right)_{1b}\right|^2\left[F\left(m_Z^2,\mu_b^2\right)-F\left(m_W^2,\mu_b^2\right)\right]\\
&-3\left[F\left(m_Z^2,m_h^2\right)-F\left(m_W^2,m_h^2\right)\right]\Big\}\,.
\end{aligned}
\tag{25}
$$

Our general results have been checked to be consistent with specific results for $\Delta\rho$ in a few models. These include the results for both the CP conserving version [5, 13] and the CP non-conserving version [16] of the 2HDM.[6] It has also been checked against a model containing one doublet and one scalar singlet [23].

3 Derivation of the result

This section contains the derivation of equation (23). It may be skipped by those who are not interested in the details of that derivation.

3.1 The Lagrangian

We use the conventions of [22]. The covariant derivative of the doublets is

$$
D_\mu \phi_k = \begin{pmatrix} \partial_\mu \varphi_k^+ - i\,\dfrac{g}{\sqrt{2}}\,W_\mu^+\varphi_k^0 + i\,\dfrac{g\left(s_W^2-c_W^2\right)}{2c_W}\,Z_\mu\varphi_k^+ + ieA_\mu\varphi_k^+ \\ \partial_\mu\varphi_k^0 - i\,\dfrac{g}{\sqrt{2}}\,W_\mu^-\varphi_k^+ + i\,\dfrac{g}{2c_W}\,Z_\mu\varphi_k^0 \end{pmatrix}
\tag{26}
$$

and the covariant derivative of the charged singlets is

$$
D_\mu\chi_j^+ = \partial_\mu\chi_j^+ + i\,\frac{g s_W^2}{c_W}\,Z_\mu\chi_j^+ + ieA_\mu\chi_j^+.
\tag{27}
$$

The covariant derivative of the neutral singlets is, of course, just identical with their ordinary derivative. We use the unitarity of $\tilde{U}$ in equation (19), in particular

$$
\left(T^\dagger T\right)_{a'a} = \delta_{a'a} - \left(U^\dagger U\right)_{a'a}.
\tag{28}
$$

We also use the orthogonality of $\tilde{V}$ in equation (20) to arrive at the gauge-kinetic Lagrangian

$$
\begin{aligned}
&\sum_{k=1}^{n_d}\left(D^\mu\phi_k\right)^\dagger\left(D_\mu\phi_k\right)+\sum_{j=1}^{n_c}\left(D^\mu\chi_j^-\right)\left(D_\mu\chi_j^+\right)+\frac{1}{2}\sum_{l=1}^{n_n}\left(\partial^\mu\chi_l^0\right)\left(\partial_\mu\chi_l^0\right)\\
=\ &\sum_{a=1}^{n}\left(\partial^\mu S_a^-\right)\left(\partial_\mu S_a^+\right)+\frac{1}{2}\sum_{b=1}^{m}\left(\partial^\mu S_b^0\right)\left(\partial_\mu S_b^0\right)
\end{aligned}
\tag{29a}
$$

[6] There is some discrepancy between our result and the one presented in Section 4 of [11].

$$+m_W^2 W^{\mu-} W_\mu^+ + m_Z^2 \frac{Z^\mu Z_\mu}{2} \tag{29b}$$

$$+ i m_W \sum_{a=1}^{n} \left[W_\mu^- \left(\omega^\dagger U \right)_a \partial^\mu S_a^+ - W_\mu^+ \left(U^\dagger \omega \right)_a \partial^\mu S_a^- \right] \tag{29c}$$

$$+ m_Z Z_\mu \sum_{b=1}^{m} \operatorname{Im} \left(\omega^\dagger V \right)_b \partial^\mu S_b^0 \tag{29d}$$

$$- \left(e m_W A^\mu + g s_W^2 m_Z Z^\mu \right) \sum_{a=1}^{n} \left[\left(\omega^\dagger U \right)_a W_\mu^- S_a^+ + \left(U^\dagger \omega \right)_a W_\mu^+ S_a^- \right] \tag{29e}$$

$$+ i e A_\mu \sum_{a=1}^{n} \left(S_a^+ \partial^\mu S_a^- - S_a^- \partial^\mu S_a^+ \right) \tag{29f}$$

$$+ i \frac{g}{2 c_W} Z_\mu \sum_{a,a'=1}^{n} \left[2 s_W^2 \delta_{aa'} - \left(U^\dagger U \right)_{a'a} \right] \left(S_a^+ \partial^\mu S_{a'}^- - S_{a'}^- \partial^\mu S_a^+ \right) \tag{29g}$$

$$+ \frac{g}{2 c_W} Z_\mu \sum_{b=1}^{m-1} \sum_{b'=b+1}^{m} \operatorname{Im} \left(V^\dagger V \right)_{bb'} \left(S_b^0 \partial^\mu S_{b'}^0 - S_{b'}^0 \partial^\mu S_b^0 \right) \tag{29h}$$

$$\begin{aligned} & + i \frac{g}{2} \sum_{a=1}^{n} \sum_{b=1}^{m} \left[\left(U^\dagger V \right)_{ab} W_\mu^+ \left(S_a^- \partial^\mu S_b^0 - S_b^0 \partial^\mu S_a^- \right) \right. \\ & \left. + \left(V^\dagger U \right)_{ba} W_\mu^- \left(S_b^0 \partial^\mu S_a^+ - S_a^+ \partial^\mu S_b^0 \right) \right] \end{aligned} \tag{29i}$$

$$+ g \left(m_W W_\mu^+ W^{\mu-} + \frac{m_Z}{c_W} \frac{Z_\mu Z^\mu}{2} \right) \sum_{b=1}^{m} S_b^0 \operatorname{Re} \left(\omega^\dagger V \right)_b \tag{29j}$$

$$- \left(\frac{eg}{2} A^\mu + \frac{g^2 s_W^2}{2 c_W} Z^\mu \right) \sum_{a=1}^{n} \sum_{b=1}^{m} S_b^0 \left[\left(U^\dagger V \right)_{ab} W_\mu^+ S_a^- + \left(V^\dagger U \right)_{ba} W_\mu^- S_a^+ \right] \tag{29k}$$

$$+ \left(\frac{g^2}{4} W^{\mu-} W_\mu^+ + \frac{g^2}{4 c_W^2} \frac{Z^\mu Z_\mu}{2} \right) \sum_{b,b'=1}^{m} \left(V^\dagger V \right)_{b'b} S_{b'}^0 S_b^0 \tag{29l}$$

$$+ \frac{g^2}{2} W^{\mu-} W_\mu^+ \sum_{a,a'=1}^{n} \left(U^\dagger U \right)_{a'a} S_{a'}^- S_a^+ \tag{29m}$$

$$+ 2 e^2 \frac{A^\mu A_\mu}{2} \sum_{a=1}^{n} S_a^- S_a^+ \tag{29n}$$

$$+ \frac{eg}{c_W} A^\mu Z_\mu \sum_{a,a'=1}^{n} \left[2 s_W^2 \delta_{aa'} - \left(U^\dagger U \right)_{a'a} \right] S_{a'}^- S_a^+ \tag{29o}$$

$$+ \frac{g^2}{2 c_W^2} \frac{Z^\mu Z_\mu}{2} \sum_{a,a'=1}^{n} \left[4 s_W^4 \delta_{aa'} + \left(1 - 4 s_W^2 \right) \left(U^\dagger U \right)_{a'a} \right] S_{a'}^- S_a^+ . \tag{29p}$$

In lines (29c)–(29e) and (29j) we have used an n_d-vector ω defined by $\omega_k \equiv v_k / v$. By identifying lines (29c) and (29d) with the usual terms [22] mixing the $W^\pm$ and Z^0 gauge

bosons with the $G^\pm$ and G^0 Goldstone bosons, respectively,

$$im_W \left(W_\mu^- \partial^\mu G^+ - W_\mu^+ \partial^\mu G^- \right) + m_Z Z_\mu \partial^\mu G^0,$$

we conclude that the components of the Goldstone bosons are given by [21]

$$U_{k1} = \frac{v_k}{v}, \quad \text{hence} \quad T_{j1} = 0, \tag{30}$$

$$V_{k1} = i\, \frac{v_k}{v}, \quad \text{hence} \quad R_{l1} = 0. \tag{31}$$

Therefore, we may rewrite line (29e) as

$$- \left(e m_W A^\mu + g s_W^2 m_Z Z^\mu \right) \left(W_\mu^- G^+ + W_\mu^+ G^- \right) \tag{32}$$

and line (29j) as

$$- g \left(m_W W_\mu^+ W^{\mu -} + \frac{m_Z}{c_W} \frac{Z_\mu Z^\mu}{2} \right) \sum_{b=2}^{m} S_b^0 \operatorname{Im} \left(V^\dagger V \right)_{1b}. \tag{33}$$

The sum starts at $b = 2$ because $\operatorname{Im} \left(V^\dagger V \right)_{11} = 0$.

If there are in the theory any $SU(2)$-singlet scalars $S^{\pm Q}$ with electric charge $\pm Q$ other than 0 or ± 1, then those scalars do not mix with components of the doublets. Their covariant derivative is

$$D_\mu S^{+Q} = \partial_\mu S^{+Q} + i\, \frac{g s_W^2 Q}{c_W}\, Z_\mu S^{+Q} + ieQ A_\mu S^{+Q}. \tag{34}$$

This yields, in particular, the following two interaction terms in the Lagrangian:

$$\mathcal{L} = \cdots + i\, \frac{g s_W^2 Q}{c_W}\, Z_\mu \left(S^{+Q} \partial^\mu S^{-Q} - S^{-Q} \partial^\mu S^{+Q} \right) \tag{35a}$$

$$+ \left(\frac{g s_W^2 Q}{c_W} \right)^2 Z_\mu Z^\mu S^{-Q} S^{+Q}. \tag{35b}$$

3.2 The Feynman diagrams

In our model, in the computation of the vacuum polarizations of the gauge bosons $W^\pm$ and Z^0 there are four types of Feynman diagrams involving scalar fields:

Type (a) diagrams: A scalar branches off from the gauge-boson line and loops back to *the same point* in that gauge-boson line—see figure 1(a). When the scalar is neutral, the relevant interaction terms in the Lagrangian are the ones in line (29l), for $b' = b$; but then the contribution to $\Delta\rho$ vanishes, since one obtains $\Pi_{WW}^{\mu\nu} = c_W^2 \Pi_{ZZ}^{\mu\nu}$. When the scalar is charged, the relevant terms in the Lagrangian are those in line (29m) for $\Pi_{WW}^{\mu\nu}$ and line (29p) for $\Pi_{ZZ}^{\mu\nu}$, in both cases for $a' = a$.

Type (b) diagrams: The gauge-boson line splits into two scalar lines which later reunite to form a new gauge-boson line—see figure 1(b). The relevant terms in the Lagrangian are those in line (29i) for $\Pi_{WW}^{\mu\nu}$, and those in lines (29g) and (29h) for $\Pi_{ZZ}^{\mu\nu}$.

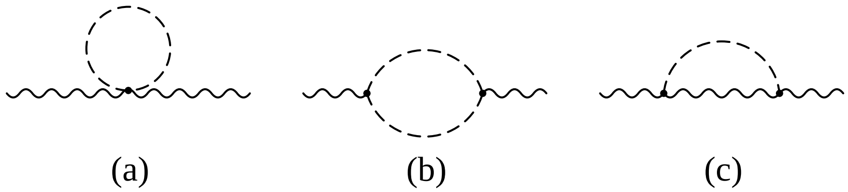

Figure 1: Three types of Feynman diagrams occurring in the calculation of the vacuum polarizations.

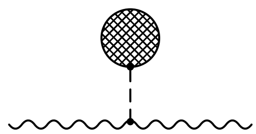

Figure 2: Tadpole diagrams which do not contribute to $\Delta\rho$.

Type (c) diagrams: A neutral scalar branches off from the gauge-boson line and loops to *a later point* in that gauge-boson line—see figure 1(c). The interaction terms in the Lagrangian responsible for these Feynman diagrams are those in expression (33).

Type (d) diagrams: A neutral scalar branches off, with zero momentum, from the gauge-boson line, and produces a loop of some stuff—see figure 2. These "tadpole" Feynman diagrams originate from the interaction terms in expression (33). They yield a vanishing contribution to $\Delta\rho$ since one obtains $\Pi_{WW}^{\mu\nu} = c_W^2 \Pi_{ZZ}^{\mu\nu}$. Hence we may omit the tadpole diagrams altogether.

3.3 Computation of the loop diagrams

We use dimensional regularization in the computation of the Feynman diagrams. The dimension of space–time is d. An unphysical mass μ is used to keep the dimension of each integral unchanged when d varies. We define the divergent quantity

$$\mathrm{div} \equiv \frac{2}{4-d} - \gamma + 1 + \ln\left(4\pi\mu^2\right),$$

where γ is Euler's constant. In the computation of type (a) Feynman diagrams the relevant momentum integral is

$$\mu^{4-d} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{g^{\mu\nu}}{k^2 - A + i\varepsilon} = \frac{i g^{\mu\nu}}{16\pi^2} A \left(\mathrm{div} - \ln A\right), \tag{36}$$

where A is the mass squared of the scalar particle in the loop. In order to compute the type (b) and type (c) Feynman diagrams we need first to introduce a Feynman parameter x, which is later integrated over from $x = 0$ to $x = 1$. For type (b) diagrams we have

$$\mu^{4-d} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \int_0^1 \mathrm{d}x \, \frac{4\, k^\mu k^\nu}{\left[k^2 - Ax - B\left(1-x\right) + i\varepsilon\right]^2} \quad = \quad \frac{i g^{\mu\nu}}{16\pi^2} \left[A \left(\mathrm{div} - \ln A\right)\right.$$

$$+B\left(\mathrm{div}-\ln B\right)+F\left(A,B\right)\right], \quad (37)$$

where A and B are the masses squared of the scalars in the loop, and the four-momentum q^μ of the external gauge-boson line is taken to obey $q^2=0$. Notice the presence of terms of the form $A\left(\mathrm{div}-\ln A\right)$ in both diagrams of types (a) and (b); we shall soon see that those terms cancel out in the computation of $\Delta\rho$, leaving only the F functions from the type (b) diagrams. For type (c) diagrams the relevant integral is

$$\mu^{4-d}\int\frac{\mathrm{d}^d k}{\left(2\pi\right)^d}\int_0^1\mathrm{d}x\,\frac{g^{\mu\nu}}{\left[k^2-Ax-B\left(1-x\right)+i\varepsilon\right]^2} = \frac{ig^{\mu\nu}}{16\pi^2}\,\frac{1}{A}\left[A\left(\mathrm{div}-\ln A\right)-\frac{A+B}{2}+F\left(A,B\right)\right]. \quad (38)$$

This integral is symmetric under the interchange of A and B; equation (38) presents a seemingly asymmetric form, but it is in fact symmetric. The reason for expressing the integral in this way is that, due to cancellations, only the terms $F\left(A,B\right)$ survive in the end.

3.4 The contributions to $\Delta\rho$ from diagrams of types (a) and (b)

Using (29m) and (36), we see that the contribution to $A_{WW}\left(q^2\right)$ of type (a) Feynman diagrams with charged scalars in the loop is

$$A^{(a)}_{WW}\left(q^2\right)=-\frac{g^2}{32\pi^2}\sum_{a=1}^{n}\left(U^\dagger U\right)_{aa} m_a^2\left(\mathrm{div}-\ln m_a^2\right). \quad (39)$$

In the same way, using (29p),

$$A^{(a)}_{ZZ}\left(q^2\right)=-\frac{g^2}{32\pi^2 c_W^2}\sum_{a=1}^{n}\left[4s_W^4+\left(1-4s_W^2\right)\left(U^\dagger U\right)_{aa}\right] m_a^2\left(\mathrm{div}-\ln m_a^2\right). \quad (40)$$

Proceeding to the type (b) Feynman diagrams, from (29i) and (37) we find that

$$\begin{aligned}
A^{(b)}_{WW}\left(0\right) &= \frac{g^2}{64\pi^2}\sum_{a=1}^{n}\sum_{b=1}^{m}\left(U^\dagger V\right)_{ab}\left(V^\dagger U\right)_{ba}\left[m_a^2\left(\mathrm{div}-\ln m_a^2\right)\right. \\
&\quad \left. +\mu_b^2\left(\mathrm{div}-\ln \mu_b^2\right)+F\left(m_a^2,\mu_b^2\right)\right] \\
&= \frac{g^2}{64\pi^2}\left[2\sum_{a=1}^{n}\left(U^\dagger U\right)_{aa} m_a^2\left(\mathrm{div}-\ln m_a^2\right)\right. && \text{(41a)} \\
&\quad +\sum_{b=1}^{m}\left(V^\dagger V\right)_{bb}\mu_b^2\left(\mathrm{div}-\ln \mu_b^2\right) && \text{(41b)} \\
&\quad \left.+\sum_{a=1}^{n}\sum_{b=1}^{m}\left|\left(U^\dagger V\right)_{ab}\right|^2 F\left(m_a^2,\mu_b^2\right)\right]. && \text{(41c)}
\end{aligned}$$

We have used

$$\sum_{a=1}^{n}\left(U^\dagger V\right)_{ab}\left(V^\dagger U\right)_{ba}=\left(V^\dagger V\right)_{bb}, \quad (42)$$

which follows from the unitarity of $\tilde{U}$, i.e. from [21]

$$UU^\dagger = 1_{n_d \times n_d}. \tag{43}$$

We have also used

$$\sum_{b=1}^{m} \left(U^\dagger V\right)_{ab} \left(V^\dagger U\right)_{ba} = 2 \left(U^\dagger U\right)_{aa}, \tag{44}$$

which follows from the orthogonality of $\tilde{V}$, i.e. from [21]

$$\begin{array}{rclcl} \mathrm{Re}\, V\ \mathrm{Re} V^T & = & \mathrm{Im}\, V\ \mathrm{Im} V^T & = & 1_{n_d \times n_d}, \\ \mathrm{Re}\, V\ \mathrm{Im} V^T & = & \mathrm{Im}\, V\ \mathrm{Re} V^T & = & 0_{n_d \times n_d}. \end{array} \tag{45}$$

Considering now the self-energy of the Z^0 boson, we find

$$\begin{aligned} A_{ZZ}^{(b)}(0) &= \frac{g^2}{64\pi^2 c_W^2} \left\{ \sum_{a,a'=1}^{n} \left[2 s_W^2 \delta_{aa'} - \left(U^\dagger U\right)_{a'a}\right] \left[2 s_W^2 \delta_{aa'} - \left(U^\dagger U\right)_{aa'}\right] \right. \\ & \quad \times \left[m_a^2 \left(\mathrm{div} - \ln m_a^2\right) + m_{a'}^2 \left(\mathrm{div} - \ln m_{a'}^2\right) + F\left(m_a^2, m_{a'}^2\right)\right] \\ & \quad + \sum_{b=1}^{m-1} \sum_{b'=b+1}^{m} \left[\mathrm{Im}\left(V^\dagger V\right)_{bb'}\right]^2 \\ & \quad \left. \times \left[\mu_b^2 \left(\mathrm{div} - \ln \mu_b^2\right) + \mu_{b'}^2 \left(\mathrm{div} - \ln \mu_{b'}^2\right) + F\left(\mu_b^2, \mu_{b'}^2\right)\right] \right\} \\ &= \frac{g^2}{64\pi^2 c_W^2} \left\{ 2 \sum_{a=1}^{n-1} \sum_{a'=a+1}^{n} \left| \left(U^\dagger U\right)_{aa'} \right|^2 F\left(m_a^2, m_{a'}^2\right) \right. && \text{(46a)} \\ & \quad + 2 \sum_{a=1}^{n} \left[4 s_W^4 + \left(1 - 4 s_W^2\right) \left(U^\dagger U\right)_{aa}\right] m_a^2 \left(\mathrm{div} - \ln m_a^2\right) && \text{(46b)} \\ & \quad + \sum_{b=1}^{m-1} \sum_{b'=b+1}^{m} \left[\mathrm{Im}\left(V^\dagger V\right)_{bb'}\right]^2 F\left(\mu_b^2, \mu_{b'}^2\right) && \text{(46c)} \\ & \quad \left. + \sum_{b=1}^{m} \left(V^\dagger V\right)_{bb} \mu_b^2 \left(\mathrm{div} - \ln \mu_b^2\right) \right\}. && \text{(46d)} \end{aligned}$$

We have used

$$\sum_{b'=1}^{m} \left[\mathrm{Im}\left(V^\dagger V\right)_{bb'}\right]^2 = \left(V^\dagger V\right)_{bb}, \tag{47}$$

which follows from equations (45).

Putting everything together, we see that

the $A_{WW}^{(a)}(q^2)$ of equation (39) cancels out the line (41a) of $A_{WW}^{(b)}(0)$;

the $A_{ZZ}^{(a)}(q^2)$ of equation (40) cancels out the line (46b) of $A_{ZZ}^{(b)}(0)$;

the line (41b) of $A_{WW}^{(b)}(0)$ cancels out the line (46d) of $A_{ZZ}^{(b)}(0)$ in the subtraction $A_{WW} - c_W^2 A_{ZZ}$.

In this way we finally obtain

$$
\begin{aligned}
A_{WW}^{(a+b)}(0) - c_W^2 A_{ZZ}^{(a+b)}(0) &= \frac{g^2}{64\pi^2} \left\{ \sum_{a=1}^{n} \sum_{b=1}^{m} \left| \left(U^\dagger V\right)_{ab} \right|^2 F\left(m_a^2, \mu_b^2\right) \right. && \text{(48a)} \\
&\quad -2 \sum_{a=1}^{n-1} \sum_{a'=a+1}^{n} \left| \left(U^\dagger U\right)_{aa'} \right|^2 F\left(m_a^2, m_{a'}^2\right) && \text{(48b)} \\
&\quad \left. - \sum_{b=1}^{m-1} \sum_{b'=b+1}^{m} \left[\mathrm{Im} \left(V^\dagger V\right)_{bb'} \right]^2 F\left(\mu_b^2, \mu_{b'}^2\right) \right\}. && \text{(48c)}
\end{aligned}
$$

The positive term (48a) originates from $A_{WW}^{(b)}$ while the negative terms (48b) and (48c) come from $A_{ZZ}^{(b)}$.

If there are in the electroweak theory any scalar $SU(2)$ singlets with electric charges other than 0 or ± 1, then the relevant terms in the Lagrangian are those in equation (35). The term (35b) generates a type (a) Feynman diagram which exactly cancels the type (b) Feynman diagram generated by the term (35a).[7] Thus, scalar $SU(2)$ singlets with electric charge different from 0 and ± 1 do not affect $\Delta\rho$ at all.

The sums in equation (48) include contributions from the Goldstone bosons $G^\pm = S_1^\pm$ and $G^0 = S_1^0$. These Goldstone bosons have unphysical masses m_1 and μ_1, respectively, which are arbitrary in a 't Hooft gauge. The terms which depend on those masses are, explicitly,

$$
\begin{aligned}
& \left| \left(U^\dagger V\right)_{11} \right|^2 F\left(m_1^2, \mu_1^2\right) && \text{(49a)} \\
& + \sum_{b=2}^{m} \left| \left(U^\dagger V\right)_{1b} \right|^2 F\left(m_1^2, \mu_b^2\right) && \text{(49b)} \\
& + \sum_{a=2}^{n} \left| \left(U^\dagger V\right)_{a1} \right|^2 F\left(m_a^2, \mu_1^2\right) && \text{(49c)} \\
& -2 \sum_{a=2}^{n} \left| \left(U^\dagger U\right)_{1a} \right|^2 F\left(m_1^2, m_a^2\right) && \text{(49d)} \\
& - \sum_{b=2}^{m} \left[\mathrm{Im} \left(V^\dagger V\right)_{1b} \right]^2 F\left(\mu_1^2, \mu_b^2\right). && \text{(49e)}
\end{aligned}
$$

One may eliminate some of these terms by using equations (30) and (31). Indeed, $\left(U^\dagger U\right)_{1a} = -\left(T^\dagger T\right)_{1a} = 0$ when $a \neq 1$, because $T_{j1} = 0$ for any j; also, $\left(U^\dagger V\right)_{a1} = i\left(U^\dagger U\right)_{a1} = 0$ for $a \neq 1$. Therefore, the terms (49c) and (49d) vanish. In the term (49a), $\left(U^\dagger V\right)_{11} = i$. In the term (49b) one may write

$$
\left(U^\dagger V\right)_{1b} = i\left(V^\dagger V\right)_{1b} = -\mathrm{Im}\left(V^\dagger V\right)_{1b} \Leftarrow b \neq 1, \tag{50}
$$

since $\mathrm{Re}\left(V^\dagger V\right)_{1b} = \left(\mathrm{Re}\, V^T\, \mathrm{Re}\, V + \mathrm{Im}\, V^T\, \mathrm{Im}\, V\right)_{1b} = -\left(R^T R\right)_{1b} = 0$. In this way, the

[7] This cancellation is analogous to the one between equation (40) and line (46b).

terms (49) are reduced to

$$F\left(m_1^2,\mu_1^2\right)+\sum_{b=2}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{1b}\right]^2\left[F\left(m_1^2,\mu_b^2\right)-F\left(\mu_1^2,\mu_b^2\right)\right]. \tag{51}$$

The term $F\left(m_1^2,\mu_1^2\right)$ is independent of the number of scalar doublets and singlets, hence it is eliminated when one subtracts the SM result from the Multi-Higgs-doublet-model one. The other terms in the expression (51) are cancelled out by the diagrams of type (c), as we shall see next.

3.5 The contributions to $\Delta\rho$ from diagrams of type (c)

To compensate for the unphysical masses of the Goldstone bosons, the propagators of gauge bosons $W^\pm$ and Z^0 with four-momentum k^μ are, in a 't Hooft gauge,

$$-\frac{k_\mu k_\nu}{m_W^2}\,\frac{i}{k^2-m_1^2}+\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_W^2}\right)\frac{i}{k^2-m_W^2}, \tag{52}$$

$$-\frac{k_\mu k_\nu}{m_Z^2}\,\frac{i}{k^2-\mu_1^2}+\left(-g_{\mu\nu}+\frac{k_\mu k_\nu}{m_Z^2}\right)\frac{i}{k^2-m_Z^2}, \tag{53}$$

respectively, i.e. they contain a piece with a pole on the unphysical masses squared m_1^2 and μ_1^2, respectively.

Using these propagators to compute the type (c) Feynman diagrams, one obtains

$$\begin{aligned}
A_{WW}^{(c)}(0) &= \frac{g^2}{64\pi^2}\sum_{b=2}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{1b}\right]^2\left[-m_1^2\left(\mathrm{div}-\ln m_1^2\right)-3m_W^2\left(\mathrm{div}-\ln m_W^2\right)\right.\\
&\quad\left.+2\left(m_W^2+\mu_b^2\right)-F\left(m_1^2,\mu_b^2\right)-3F\left(m_W^2,\mu_b^2\right)\right], && (54)\\
A_{ZZ}^{(c)}(0) &= \frac{g^2}{64\pi^2 c_W^2}\sum_{b=2}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{1b}\right]^2\left[-\mu_1^2\left(\mathrm{div}-\ln \mu_1^2\right)-3m_Z^2\left(\mathrm{div}-\ln m_Z^2\right)\right.\\
&\quad\left.+2\left(m_Z^2+\mu_b^2\right)-F\left(\mu_1^2,\mu_b^2\right)-3F\left(m_Z^2,\mu_b^2\right)\right]. && (55)
\end{aligned}$$

The factors 3 originate in a partial cancellation between the contributions from the pieces $-g_{\mu\nu}$ and $k_\mu k_\nu/m_V^2$ in the propagator of the gauge boson V, the former contribution being four times larger than, and with opposite sign relative to, the latter one, cf. equations (37) and (38). Performing the subtraction relevant for $\Delta\rho$, one obtains

$$\begin{aligned}
A_{WW}^{(c)}(0)-c_W^2 A_{ZZ}^{(c)}(0) &= \frac{g^2}{64\pi^2}\sum_{b=2}^{m}\left[\mathrm{Im}\left(V^\dagger V\right)_{1b}\right]^2\\
&\quad\times\left[-m_1^2\left(\mathrm{div}-\ln m_1^2\right)+\mu_1^2\left(\mathrm{div}-\ln \mu_1^2\right)\right. && (56a)\\
&\quad-3m_W^2\left(\mathrm{div}-\ln m_W^2\right)+3m_Z^2\left(\mathrm{div}-\ln m_Z^2\right) && (56b)\\
&\quad+2\left(m_W^2-m_Z^2\right) && (56c)\\
&\quad-F\left(m_1^2,\mu_b^2\right)+F\left(\mu_1^2,\mu_b^2\right) && (56d)\\
&\quad\left.-3F\left(m_W^2,\mu_b^2\right)+3F\left(m_Z^2,\mu_b^2\right)\right]. && (56e)
\end{aligned}$$

The terms (56a)–(56c) are independent of the number of scalar doublets. They disappear when one subtracts the Standard-Model result from the multi-Higgs-doublet-model one, since

$$\sum_{b=2}^{m} \left[\mathrm{Im} \left(V^\dagger V \right)_{1b} \right]^2 = \left(V^\dagger V \right)_{11} = 1. \tag{57}$$

The terms (56d), which involve the masses of the Goldstone bosons, cancel out the terms in (51), except the first one, which is cancelled by the subtraction of the SM result.

We have thus finished the derivation of equation (23) for $\Delta\rho$.

4 The 2HDM and the Zee model

In this section we give, as examples of the application of our general formulae, the expressions for $\Delta\rho$ in the 2HDM and also in the model of Zee [24] for the radiative generation of neutrino masses, which has one singly charged $SU(2)$ singlet together with the two doublets.

In the study of the 2HDM it is convenient to use the so-called "Higgs basis," in which only the first Higgs doublet has a vacuum expectation value. In that basis,

$$\phi_1 = \left(\begin{array}{c} G^+ \\ \left(v + H + i G^0 \right) / \sqrt{2} \end{array} \right), \quad \phi_2 = \left(\begin{array}{c} S_2^+ \\ \left(R + i I \right) / \sqrt{2} \end{array} \right). \tag{58}$$

Here, $G^+ \equiv S_1^+$ and $G^0 \equiv S_1^0$ are the Goldstone bosons, while S_2^+ is the physical charged scalar, which has mass m_2. Thus, the matrix U, which connects the charged components of ϕ_1 and ϕ_2 to the eigenstates of mass, is in the Higgs basis of the 2HDM equal to the unit matrix. On the other hand, H, R and I, which are real fields, must be rotated through a 3×3 orthogonal matrix O to obtain the three physical neutral fields $S^0_{2,3,4}$:

$$\left(\begin{array}{c} H \\ R \\ I \end{array} \right) = O \left(\begin{array}{c} S_2^0 \\ S_3^0 \\ S_4^0 \end{array} \right). \tag{59}$$

Without lack of generality we choose $\det O = +1$. Thus, the 2×4 matrix V, defined through

$$\left(\begin{array}{c} H + i G^0 \\ R + i I \end{array} \right) = V \left(\begin{array}{c} G^0 \\ S_2^0 \\ S_3^0 \\ S_4^0 \end{array} \right), \tag{60}$$

is

$$V = \left(\begin{array}{cccc} i & O_{11} & O_{12} & O_{13} \\ 0 & O_{21} + i O_{31} & O_{22} + i O_{32} & O_{23} + i O_{33} \end{array} \right). \tag{61}$$

Therefore,

$$V^\dagger V = \left(\begin{array}{cccc} 1 & -i O_{11} & -i O_{12} & -i O_{13} \\ i O_{11} & 1 & i O_{13} & -i O_{12} \\ i O_{12} & -i O_{13} & 1 & i O_{11} \\ i O_{13} & i O_{12} & -i O_{11} & 1 \end{array} \right). \tag{62}$$

The value of $\Delta\rho$ in the 2HDM is therefore, using our formula in equation (25),

$$\begin{aligned}\Delta\rho &= \frac{g^2}{64\pi^2 m_W^2}\left\{\sum_{b=2}^{4}\left(1-O_{1\,b-1}^2\right)F\left(m_2^2,\mu_b^2\right)\right.\\ &\quad -O_{13}^2 F\left(\mu_2^2,\mu_3^2\right)-O_{12}^2 F\left(\mu_2^2,\mu_4^2\right)-O_{11}^2 F\left(\mu_3^2,\mu_4^2\right)\\ &\quad \left.+3\sum_{b=2}^{4} O_{1\,b-1}^2\left[F\left(m_Z^2,\mu_b^2\right)-F\left(m_W^2,\mu_b^2\right)-F\left(m_Z^2,m_h^2\right)+F\left(m_W^2,m_h^2\right)\right]\right\}, \end{aligned} \tag{63}$$

where $\mu_{2,3,4}$ denote the the masses of $S^0_{2,3,4}$, respectively, while m_h is the mass of the Higgs boson of the SM. Equation (63) reproduces, in a somewhat simplified form, the result for $\Delta\rho$ in the 2HDM previously given in [16].

A special case of the 2HDM is the model with one "dark" scalar doublet. This means that a second doublet is added to the SM, but that doublet has no VEV and it does not mix with the standard Higgs doublet [19]. We should then identify H with the usual Higgs particle. Thus, $O_{11}=1$ and $\mu_2=m_h$. Equation (63) then simplifies to [25, 20]

$$\Delta\rho = \frac{g^2}{64\pi^2 m_W^2}\left[\sum_{b=3}^{4} F\left(m_2^2,\mu_b^2\right)-F\left(\mu_3^2,\mu_4^2\right)\right]. \tag{64}$$

This quantity is small if the three masses m_2, μ_3 and μ_4 are close together. Notice that in this case of a "dark" scalar doublet there are no vector–vector–scalar couplings involving the additional doublet, hence $\Delta\rho$ stems exclusively from type (a) and type (b) Feynman diagrams.

In the model of Zee there is, besides the two scalar $SU(2)$ doublets

$$\phi_1 = \begin{pmatrix} G^+ \\ \left(v+H+iG^0\right)/\sqrt{2}\end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H^+ \\ \left(R+iI\right)/\sqrt{2}\end{pmatrix}, \tag{65}$$

also one scalar $SU(2)$ singlet χ^+ with unit electric charge. Therefore there is a 2×2 unitary matrix K such that

$$\begin{pmatrix} H^+ \\ \chi^+ \end{pmatrix} = K \begin{pmatrix} S_2^+ \\ S_3^+ \end{pmatrix}, \tag{66}$$

where S_2^+ and S_3^+ are the physical charged scalars, which have masses m_2 and m_3, respectively. So, now the matrix U of equation (15) is

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & K_{11} & K_{12} \end{pmatrix}, \tag{67}$$

so that

$$U^\dagger U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \left|K_{11}\right|^2 & K_{11}^* K_{12} \\ 0 & K_{11} K_{12}^* & \left|K_{12}\right|^2 \end{pmatrix}. \tag{68}$$

Equations (61) and (62) retain their validity, and

$$U^\dagger V = \begin{pmatrix} i & O_{11} & O_{12} & O_{13} \\ 0 & K_{11}^* \left(O_{21} + iO_{31}\right) & K_{11}^* \left(O_{22} + iO_{32}\right) & K_{11}^* \left(O_{23} + iO_{33}\right) \\ 0 & K_{12}^* \left(O_{21} + iO_{31}\right) & K_{12}^* \left(O_{22} + iO_{32}\right) & K_{12}^* \left(O_{23} + iO_{33}\right) \end{pmatrix}. \tag{69}$$

Therefore, using our general formula (23) for $\Delta\rho$, we see that, in the model of Zee,

$$\begin{aligned} \Delta\rho \ = \ & \frac{g^2}{64\pi^2 m_W^2} \left\{ \sum_{b=2}^{4} \left(1 - O_{1\,b-1}^2\right) \sum_{a=2}^{3} \left|K_{1\,a-1}\right|^2 F\left(m_a^2, \mu_b^2\right) \right. \\ & -2 \left|K_{11} K_{12}\right|^2 F\left(m_2^2, m_3^2\right) \\ & -O_{13}^2 F\left(\mu_2^2, \mu_3^2\right) - O_{12}^2 F\left(\mu_2^2, \mu_4^2\right) - O_{11}^2 F\left(\mu_3^2, \mu_4^2\right) \\ & \left. +3 \sum_{b=2}^{4} O_{1\,b-1}^2 \left[F\left(m_Z^2, \mu_b^2\right) - F\left(m_W^2, \mu_b^2\right) - F\left(m_Z^2, m_h^2\right) + F\left(m_W^2, m_h^2\right)\right] \right\}. \end{aligned} \tag{70}$$

5 Summary

In this paper we have derived the formula for the parameter $\Delta\rho$, as defined in equation (4), in an extension of the Standard Model characterized by an arbitrary number of scalar $SU(2)$ doublets (with hypercharge $\pm 1/2$) and singlets (with arbitrary hypercharges). Our formalism is completely general, using only the masses of the scalars and their mixing matrices, which ensures that our formulae are always applicable. The computation has been carried out in a general R_ξ gauge, thereby demonstrating that the final result is independent of the masses of the unphysical scalars. We have also explicitly demonstrated that all infinities cancel out in the final result for $\Delta\rho$. In order to ease the consultation of this paper, the formulae for $\Delta\rho$ given in Section 2 have been completely separated from their derivation presented in Section 3. Our results can be applied either to check the viability of a model or to constrain its parameter space, by comparing the $\Delta\rho$, calculated in that model, with numerical bounds on $\Delta\rho$ obtained from a fit to precision data—for instance, the bound (7) found in [4]. As an illustration of our general formulae, in Section 4 we have worked out the specific cases of the two-Higgs-doublet model, with and without one extra charged scalar singlet.

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